

## Key to MATLAB Exercise 3 – Linear Space

1.

1)  
 $\gg \mathbf{a1}=[1;0;0]; \mathbf{a2}=[0;1;1]; \mathbf{a3}=[1;0;1]; \mathbf{A}=[\mathbf{a1}, \mathbf{a2}, \mathbf{a3}]$   
 $\mathbf{A} =$

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}$$

 $\gg \text{rank}(\mathbf{A})$  $\text{ans} = 3$ 

$\{\mathbf{a1}, \mathbf{a2}, \mathbf{a3}\}$  is spanning set for  $\mathbb{R}^{3 \times 1}$ , because the rank of A equals to 3.

Or

 $\gg \text{det}(\mathbf{A})$  $\text{ans} =$ 

1

$\{\mathbf{a1}, \mathbf{a2}, \mathbf{a3}\}$  is spanning set for  $\mathbb{R}^{3 \times 1}$ , for the determinant of A is not zero.

2)

 $\gg \mathbf{a1}=[1;0;0]; \mathbf{a2}=[0;1;1]; \mathbf{a3}=[1;0;1]; \mathbf{a4}=[1;2;3]; \mathbf{A}=[\mathbf{a1}, \mathbf{a2}, \mathbf{a3}, \mathbf{a4}]; \text{rref}(\mathbf{A})$  $\text{ans} =$ 

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{matrix}$$

The result shows that  $\{\mathbf{a1}, \mathbf{a2}, \mathbf{a3}\}$  is spanning set for  $\mathbb{R}^{3 \times 1}$ .

Or

 $\gg \mathbf{A1}=\mathbf{A}(:, 1:3)$  $\mathbf{A1} =$ 

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}$$

 $\gg \text{rank}(\mathbf{A1})$  $\text{ans} =$ 

3

 $\gg \text{rank}(\mathbf{A})$  $\text{ans} =$ 

3

The result shows that  $\{\mathbf{a1}, \mathbf{a2}, \mathbf{a3}\}$  is spanning set for  $\mathbb{R}^{3 \times 1}$ .

3)

 $\gg \mathbf{a1}=[2;1;-2]; \mathbf{a2}=[3;2;-2]; \mathbf{a3}=[2;2;0]; \mathbf{A}=[\mathbf{a1}, \mathbf{a2}, \mathbf{a3}]$  $\mathbf{A} =$ 

$$\begin{matrix} 2 & 3 & 2 \\ 1 & 2 & 2 \\ -2 & -2 & 0 \end{matrix}$$

```
>> rref(A)
ans =
1     0    -2
0     1     2
0     0     0
```

They are not spanning sets for  $\mathbb{R}^{3 \times 1}$ , because there are less than 3 independent vectors.

Or

```
>> det(A)
ans =
0
```

They are not spanning sets for  $\mathbb{R}^{3 \times 1}$ , because the matrix A, which is formed by the corresponding vectors, is singular.

4)

```
>> A=[2 -1 4; 1 -1 2; -2 2 -4];
```

```
>> rref(A)
```

```
ans =
1     -1      2
0      0      0
0      0      0
```

Or

```
>> rank(A)
```

```
ans =
1
```

They are not spanning sets for  $\mathbb{R}^{3 \times 1}$ .

2.

1)

```
>> x1=[-1;2;3]; x2=[3;4;2]; x=[2;6;6]; y=[-9;-2;5];
```

```
>> A1=[x1,x2,x];
```

```
>> rref(A1)
```

```
ans =
1     0      0
0     1      0
0     0      1
```

$x \notin \text{Span}(x_1, x_2)$ , for the vectors  $x_1, x_2, x$  are independent.

Or

```
>> rank(A1)
```

```
ans =

```

```
3
```

```
>> rank([x1, x2])
```

```
ans =

```

```
2
```

$x \notin \text{Span}(x_1, x_2)$ , for the rank of  $\{x_1, x_2\}$  is not equal to that of  $\{x_1, x_2, x\}$ .

2)

```
>> rref([x1,x2,y])
ans =
  1     0     3
  0     1    -2
  0     0     0
```

Result implies that  $y \in \text{Span}(x_1, x_2)$ .

3.

1)

```
>> x1=[1; 0; 0]; x2=[0; 0; 1]; x3=[1; 0; 1]; A=[x1, x2, x3]; r=rank(A);
>> m2=size(A, 1);      % calculate the row size of A
>> if r==m2
    disp (' They are spanning sets for P3 ');
else
    disp (' They are not spanning sets for P3' );
end
```

It is not spanning set for P<sub>3</sub>.

2)

```
>> x1=[2; 0; 0]; x2=[-2; 1; 0]; x3=[0; 1; 0]; x4=[1; 0; 2]; rank([x1, x2, x3, x4])
ans=
  3
```

$\{2, x-2, x, 2x^2 + 1\}$  is spanning set for P<sub>3</sub>.

3)

```
>> x1=[1; 1; 0]; x2=[-2; 1; 0]; x3=[3; 0; 1]; rref([x1, x2, x3])
ans =
  1     0     0
  0     1     0
  0     0     1
```

It means that  $\{x+1, x-2, x^2 + 3\}$  is spanning set for P<sub>3</sub>.

4.

1)

```
>> a1=[1 1 1]'; a2=[0 1 1]'; a3=[1 0 1]'; rank([a1, a2, a3])
ans =
  3
```

They are linearly independent.

2)

```
>> a1=[2 -1 2]'; a2=[2 2 0]'; A=[a1, a2];
>> rank(A)==size(A, 2)  %check whether the rank of A is equal to the column size of A.
ans =
  1          % 1 means "true", 0 means "false"
```

They are linearly independent.

3)

```
>> a1=[2 -1 2]'; a2=[-2 1 -2]'; a3=[4 2 -4]'; A=[a1,a2,a3]; rank(A)==size(A, 2)
```

```
ans =
```

```
0
```

They are linearly dependent.

5.

For example 2)

```
>> x1=[2; 0; 0]; x2=[-2; 1; 0]; x3=[0; 1; 0]; x4=[1; 0; 2]; rref([x1, x2, x3, x4])
```

```
ans =
```

```
1 0 1 0  
0 1 1 0  
0 0 0 1
```

$\text{Span}\{2, x-2, x, 2x^2 + 1\} = \text{Span}\{2, x-2, 2x^2 + 1\}$ .

6.

```
>> x1=[-1 2 3]'; x2=[3 4 2]'; x3=[0 10 11]';
```

1)

```
>> rank([x1,x2,x3])
```

```
ans =
```

```
2
```

Or

```
>> rank(x1, x2, x3)==size([x1, x2, x3], 2)
```

```
ans =
```

```
0
```

$x_1, x_2, x_3$  are linearly dependent.

2)

```
>> rank([x1 ,x2])
```

```
ans =
```

```
2
```

$x_1$  and  $x_2$  are linearly independent.

3) The rank of vector set  $\{x_1, x_2, x_3\}$  shows the dimension is 2.

4) They are the 2-dimension subspace of  $\mathbb{R}^{3\times 1}$ .

7.

```
>> x1=[-1 2 3]'; x2=[3 4 2]'; x3=[0 10 11]'; x4=[2 7 3]'; x5=[-1 3 2]'; rref([x1,x2,x3,x4,x5])
```

```
ans =
```

```
1.0000 0 3.0000 0 0.4194  
0 1.0000 1.0000 0 -0.6452  
0 0 0 1.0000 0.6774
```

$\{x_1, x_2, x_4\}$  forms a basis for  $\mathbb{R}^3$ .

8.

```
>> x1=[3;2]; x2=[1;3]; A=[x1 x2]; x=[10;7]; c=inv(A)*x
```

```
c =  
3.2857  
0.1429
```

9.

```
>> x1=[2; 4]; x2=[1; 3]; B=[x1, x2];  
>> Transition_matrix=B  
Transition_matrix =  
2 1  
4 3  
>> x=[10; 7];  
>> coordinates=inv(Transition_matrix)*x  
coordinates =  
11.5000  
-13.0000
```

10.

```
>> X=[x1, x2]; Y=[y1, y2];  
>> Transition_matrix=X\Y  
Transition_matrix =  
-2.5000 2.5000  
4.0000 -2.0000
```

11.

```
1)  
>> A=[1 2 3;-2 3 6;9 4 6]  
A =  
1 2 3  
-2 3 6  
9 4 6  
>> x1=A(1,:); x2=A(2,:); x3=A(3,:);  
>> y1=A(:,1); y2=A(:,2); y3=A(:,3);  
>> rref(A)  
ans =  
1 0 0  
0 1 0  
0 0 1
```

So a basis for the row space is  $\{x_1, x_2, x_3\}$ , a basis for the column space is  $\{y_1, y_2, y_3\}$ . The nullspace is  $\{0\}$ .

2)

```
>> x1=A(1,:); x2=A(2,:); x3=A(3,:); y1=A(:,1); y2=A(:,2); y3=A(:,3); y4=A(:,4);  
>> rref(A)  
ans =  
1 0 1 0  
0 1 1 0  
0 0 0 1
```

So a basis for the row space is  $\{x_1, x_2, x_3\}$ ; a basis for the column space is  $\{y_1, y_2, y_4\}$  and

a basis for the null space is  $[-1 -1 1 0]'$ .

3)

```
>> A=[-1 2 1 0;3 2 5 6;2 0 6 6];
>> x1=A(1,:); x2=A(2,:); x3=A(3,:); y1=A(:,1); y2=A(:,2); y3=A(:,3); y4=A(:,4);
>> rref(A)
ans =
    1.0000      0      0     0.7500
            0    1.0000      0         0
            0      0    1.0000     0.7500
```

So a basis for the row space is  $\{x_1, x_2, x_3\}$ ; a basis for the column space is  $\{y_1, y_2, y_3\}$ .

Notice that  $[-0.7500 \ 0 \ -0.7500 \ 1]'$  is a solution of  $Ax=0$  and  $\text{rank}(A)=3$ , then a basis for the null space is  $[-0.7500 \ 0 \ -0.7500 \ 1]'$ .

12.

```
❖ >> A10=rand(10);
>> b10=rand(10, 1);
>> tic;
>> x1=inv(A10)*b10;
>> toc
Elapsed time is 28.611000 seconds.

❖ >> tic;
>> [l,u]=lu(A10);
>> y1=inv(l)*b10;
>> x2=inv(u)*y1;
>> toc
Elapsed time is 50.012000 seconds.

❖ >> tic;
>> [q,r]=qr(A10);
>> y1=q'*b;
>> y1=q'*b10;
>> x1=inv(r)*y1;
>> toc
Elapsed time is 105.983000 seconds.
```