Key to MATLAB Exercise 3 – Linear Space

1.

1)

```
>> a1=[1;0;0]; a2=[0;1;1]; a3=[1;0;1]; A=[a1,a2,a3]
A =

     1     0     1
     0     1     0
     0     1     1
>> rank(A)
ans = 3
```

{a1,a2,a3} is spanning set for $R^{3\times 1}$, because the rank of A equals to 3.

Or

```
>> det(A)
ans =
     1
```

{a1,a2,a3} is spanning set for $R^{3\times 1}$, for the determinant of A is not zero.

2)

```
>> a1=[1;0;0]; a2=[0;1;1]; a3=[1;0;1]; a4=[1;2;3]; A=[a1,a2,a3,a4]; rref(A)
ans =
     1     0     0     0
     0     1     0     2
     0     0     1     1
```

The result shows that {a1 a2 a3} is spanning set for $R^{3\times 1}$.

Or

```
>> A1=A(:, 1:3)
A1 =
     1     0     1
     0     1     0
     0     1     1
>> rank(A1)
ans =
     3
>> rank(A)
ans =
     3
```

The result shows that {a1 a2 a3} is spanning set for $R^{3\times 1}$.

3)

```
>> a1=[2;1;-2]; a2=[3;2;-2]; a3=[2;2;0]; A=[a1,a2,a3]
A =
     2     3     2
     1     2     2
    -2    -2     0
```

```
>> rref(A)
ans =
     1     0    -2
     0     1     2
     0     0     0
```

They are not spanning sets for $R^{3\times 1}$, because there are less than 3 independent vectors.

Or

```
>> det(A)
ans =
     0
```

They are not spanning sets for $R^{3\times 1}$, because the matrix A, which is formed by the corresponding vectors, is singular.

4)

```
>> A=[2 -1 4; 1 -1 2; -2 2 -4 ];
>> rref(A)
ans =
     1    -1     2
     0     0     0
     0     0     0
```

Or

```
>> rank(A)
ans =
     1
```

They are not spanning sets for $R^{3\times 1}$.

2.

1)

```
>> x1=[-1;2;3]; x2=[3;4;2]; x=[2;6;6]; y=[-9;-2;5];
>> A1=[x1,x2,x];
>> rref(A1)
ans =
     1     0     0
     0     1     0
     0     0     1
```

$x \notin \mathrm{Span}(x_1, x_2)$, for the vectors x_1, x_2, x are independent.

Or

```
>> rank(A1)
ans =
     3
>> rank([x1, x2])
ans =
     2
```

$x \notin \mathrm{Span}(x_1, x_2)$, for the rank of $\{x_1, x_2\}$ is not equal to that of $\{x_1, x_2, x\}$.

2)

```
>> rref([x1,x2,y])
ans =
     1     0     3
     0     1    -2
     0     0     0
```

Result implies that $y \in \mathrm{Span}(x_1, x_2)$.

3.

1)

```
>> x1=[1; 0; 0]; x2=[0; 0; 1]; x3=[1; 0; 1]; A=[x1, x2, x3]; r=rank(A);
>> m2=size(A, 1);        % calculate the row size of A
>> if r==m2
     disp (' They are spanning sets for P3 ');
   else
     disp (' They are not spanning sets for P3');
   end
```

It is not spanning set for P_3.

2)

```
>> x1=[2; 0; 0]; x2=[-2; 1; 0]; x3=[0; 1; 0]; x4=[1; 0; 2]; rank([x1, x2, x3, x4])
ans=
     3
```

$\{2, x-2, x, 2x^2+1\}$ is spanning set for P_3.

3)

```
>> x1=[1; 1; 0]; x2=[-2; 1; 0]; x3=[3; 0; 1]; rref([x1, x2, x3])
ans =
     1     0     0
     0     1     0
     0     0     1
```

It means that $\{x+1, x-2, x^2+3\}$ is spanning set for P_3.

4.

1)

```
>> a1=[1 1 1]'; a2=[0 1 1]'; a3=[1 0 1]'; rank([a1, a2, a3])
ans =
     3
```

They are linearly independent.

2)

```
>> a1=[2 -1 2]'; a2=[2 2 0]'; A=[a1, a2];
>> rank(A)==size(A, 2)    %check whether the rank of A is equal to the column size of A.
ans =
     1                    % 1 means "true", 0 means "false"
```

They are linearly independent.

3)

```
>> a1=[2 -1 2]'; a2=[-2 1 -2]'; a3=[4 2 -4]'; A=[a1,a2,a3]; rank(A)==size(A, 2)
ans =
     0
```

They are linearly dependent.

5.

For example 2)

```
>> x1=[2; 0; 0]; x2=[-2; 1; 0]; x3=[0; 1; 0]; x4=[1; 0; 2]; rref([x1, x2, x3, x4])
ans =
  1     0     1     0
  0     1     1     0
  0     0     0     1
```

Span$\{2, x-2, x, 2x^2+1\}$ =Span$\{2, x-2, 2x^2+1\}$.

6.

```
>> x1=[-1 2 3]'; x2=[3 4 2]'; x3=[0 10 11]';
```

1)

```
>> rank([x1,x2,x3])
ans =
     2
```

Or

```
>> rank(x1, x2, x3)==size([x1, x2, x3], 2)
ans =
     0
```

x_1, x_2, x_3 are linearly dependent.

2)

```
>> rank([x1 ,x2])
ans =
     2
```

x_1 and x_2 are linearly independent.

3) The rank of vector set $\{x_1, x_2, x_3\}$ shows the dimension is 2.

4) They are the 2-dimension subspace of $R^{3\times 1}$.

7.

```
>> x1=[-1 2 3]'; x2=[3 4 2]'; x3=[0 10 11]'; x4=[2 7 3]'; x5=[-1 3 2]'; rref([x1,x2,x3,x4,x5])
ans =
    1.0000         0    3.0000         0    0.4194
         0    1.0000    1.0000         0   -0.6452
         0         0         0    1.0000    0.6774
```

$\{x_1, x_2, x_4\}$ forms a basis for R^3.

8.

```
>> x1=[3;2]; x2=[1;3]; A=[x1 x2]; x=[10;7]; c=inv(A)*x
```

```
c =
    3.2857
    0.1429
```

9.

```
>> x1=[2; 4]; x2=[1; 3]; B=[x1, x2];
>> Transition_matrix=B
Transition_matrix =
     2     1
     4     3
>> x=[10; 7];
>> coordinates =inv(Transition_matrix) *x
coordinates =
   11.5000
  -13.0000
```

10.

```
>> X=[x1, x2]; Y=[y1, y2];
>> Transition_matrix =X\Y
  Transition_matrix =
    -2.5000    2.5000
     4.0000   -2.0000
```

11.

1)

```
>> A=[1 2 3;-2 3 6;9 4 6]
A =
     1     2     3
    -2     3     6
     9     4     6
>> x1=A(1,:); x2=A(2,:); x3=A(3,:);
>> y1=A(:,1); y2=A(:,2); y3=A(:,3);
>> rref(A)
ans =
     1     0     0
     0     1     0
     0     0     1
```

So a basis for the row space is {x1, x2, x3}, a basis for the column space is {y1, y2, y3}. The nullspace is {0}.

2)

```
>> x1=A(1,:); x2=A(2,:); x3=A(3,:); y1=A(:,1); y2=A(:,2); y3=A(:,3); y4=A(:,4);
>> rref(A)
ans =
     1     0     1     0
     0     1     1     0
     0     0     0     1
```

So a basis for the row space is {x1, x2, x3}; a basis for the column space is {y1, y2, y4} and

a basis for the null space is [-1 -1 1 0]'.

3)

```
>> A=[-1 2 1 0;3 2 5 6;2 0 6 6 ];
>> x1=A(1,:); x2=A(2,:); x3=A(3,:); y1=A(:,1); y2=A(:,2); y3=A(:,3); y4=A(:,4);
>> rref(A)
ans =
    1.0000         0         0    0.7500
         0    1.0000         0         0
         0         0    1.0000    0.7500
```

So a basis for the row space is {x1, x2, x3}; a basis for the column space is {y1, y2, y3}. Notice that [-0.7500 0 -0.7500 1]' is a solution of Ax=0 and rank(A)=3, then a basis for the null space is [-0.7500 0 -0.7500 1]'.

12.

✧
```
>> A10=rand(10);
>> b10=rand(10, 1);
>> tic;
>> x1=inv(A10)*b10;
>> toc
Elapsed time is 28.611000 seconds.
```

✧
```
>> tic;
>> [l,u]=lu(A10);
>> y1=inv(l)*b10;
>> x2=inv(u)*y1;
>> toc
Elapsed time is 50.012000 seconds.
```

✧
```
>> tic;
>> [q,r]=qr(A10);
>> y1=q'*b;
>> y1=q'*b10;
>> x1=inv(r)*y1;
>> toc
Elapsed time is 105.983000 seconds.
```