Key to MATLAB Exercise 7 – Calculus

1.

1)

```
>> clear; syms y1 u x; y1=(1+u^2)^(1/2); u= exp(-x);
>> y1= compose(y1,u,x)
```

Or

```
>> clear; syms x; u=exp(-x); y1=(1+u^2)^(1/2)                    % no recommend
y1 =
 (1+exp(-x)^2)^(1/2)
```

2)

```
>> clear; syms y2 u v x; y2=(1+u^2)^(1/2); u =log(v); v=exp(-x);
>> u_x=compose(u,v,x); y2=compose(y2,u_x,x)
```

Or

```
>> clear; syms y2 u v x; y2=(1+u^2)^(1/2); u=log(v); v=exp(-x);
>> y2=compose(y2, compose(u, v, x))
```

Or

```
>> clear; syms x; v=exp(-x); u=log(v); y2=(1+u^2)^(1/2)          % no recommend
y2 =
 (1+log(exp(-x))^2)^(1/2)
```

3)

```
>> clear; syms y u v w x; y=(1+u^2)^(1/2); u=log(v); v=sin(w); w=exp(-x);
>> v_x=compose(v,w,x); u_x=compose(u,v_x,x); y=compose(y,u_x,x)
```

Or

```
>> clear; syms y u v w x; y=(1+u^2)^(1/2); u=log(v); v=sin(w); w=exp(-x);
>> y=compose(y, compose(u, compose(v, w, x)))
```

Or

```
>> clear; syms x; w=exp(-x); v=sin(w); u=log(v); y=(1+u^2)^(1/2)   % no recommend
y =
(1+log(sin(exp(-x)))^2)^(1/2)
```

2.

1)

```
>> clear; syms x y; y=(1+(log(sin(x)))^2)^(1/2);
>> finverse(y,x)
Warning: finverse((1+log(sin(x))^2)^(1/2)) is not unique.
> In sym.finverse at 43
ans =
asin(exp((-1+x^2)^(1/2)))
```

2)

```
>> clear; syms x u y; y=(x+log(sin(u)))^(1/2);
>> finverse(y,u)
ans =
asin(exp(-x+u^2))
```

3)

```
>> clear; syms x u y; y=(x+log(sin(u)))^(1/2);
>> finverse(y,x)
ans =
-log(sin(u))+x^2
```

3.

```
>> syms x y1 y2; y1=sin(x); y2=asin(sin(x));
>> x0=sym(pi/4); subs(y1,x0)
ans =
1/2*2^(1/2)
>> subs(y2,x0)
ans =
1/4*pi
```

Or

```
>> syms x y1 y2; y1=sin(x); y2=asin(sin(x));
>> compose(y1,pi/4)
ans =
1/2*2^(1/2)
>> compose(y2,pi/4)
ans =
1/4*pi
```

4.

1)

```
>> clear; syms n m x; y1=(tan(n*x)-sin(m*x))/x;
>> limit(y1,0)
```

Or

```
>> clear; syms n m x; y1=(tan(n*x)-sin(m*x))/x;
>> limit(y1)
ans =
n-m
```

2)

```
>> clear; syms x y; y2=(exp(x)-exp(y))/ (x-y);
>> limit(y2,x,y)
ans =
exp(y)
```

3)

```
>> clear; syms x; y3=x^3/(2*x+100);
>> limit(y3,x,+inf)
ans =
Inf
```

4)

```
>> clear; syms x; y4=x^3/sin(x);
>> limit(y4,x,-inf)
ans =
NaN
```

5)

```
>> clear; syms x; y5=(tan(x)^(tan(x)));
>> limit(y5,x,pi/4,'right')
ans =
1
```

6)

```
>> clear; syms x; y=tan(x/2);
>> limit(y,x,pi,'left')
ans =
Inf
```

5. 1)

```
>> clear; syms x n h; y=((x+h)^n-x^n)/h;
>> z=limit(y,h,0)
z =
x^n/x*n
>> simplify(z))
ans =
x^(-1+n)*n
```

2)

```
>> clear; syms x n ; y= x^n;
>> diff(y, x)
ans =
x^n/x*n
>> simplify(diff(y,x))
ans =
x^(-1+n)*n
```

6.

1)

```
>> clear; syms x g; g=(x^3-5)/(2*x^2+7);
>> subs(diff(g,x),0)
```

Or

```
>> clear; syms x g; g=(x^3-5)/(2*x^2+7);
>> compose(diff(g,x),0)
ans =
0
```

2)

```
>> clear; syms x y g; g=(x^3*y-5*y)/(2*x^2+7);
>> g_xy=diff(diff(g,x),y)
g_xy =
3*x^2/(2*x^2+7)-4*(x^3-5)/(2*x^2+7)^2*x
>> subs(g_xy, 1)
ans =
      0.5309
```

Or

```
>> clear; syms x y g; g=(x^3*y-5*y)/(2*x^2+7); g_xy=diff(diff(g,x),y)
>> compose(g_xy,1)
ans =
43/81
```

3)

```
>> clear; syms x y g; g=(x^3*y-5*y)/(2*x^2+7);
>> g_y=diff(g,y)
g_y =
  (x^3-5)/(2*x^2+7)
>> subs(g_y, [x y], [1 2])
ans =
-4/9
```

4)

```
>> clear; syms x f; f=sin(x)*sin(2*x)*sin(3*x);
>> diff(f,5)
ans =
1696*cos(x)*sin(2*x)*sin(3*x)+2192*sin(x)*cos(2*x)*sin(3*x)+2208*sin(x)*sin(2*x)*cos(3
*x)-1680*cos(x)*cos(2*x)*cos(3*x)
```

7.

1)

```
>> A=[1,5,8,-2,6,3]; B=(-1)*diff(A)
B =
     -4     -3     10     -8      3
```

2)

```
>> A=[1,5,8,-2,6,3]; C=diff(A,2)
C =
     -1    -13     18    -11
```

8.

1)

```
>> clear; syms x y; y=1/(x+1);
>> int(y)
ans =
log(x+1)
```

2)

```
>> clear; syms x y; y=1/(x+1);
>> int(y,0,1)
ans =
log(2)
```

3)

```
>> clear; syms x t y; y=1/(x+1);
>> int(y,0,t)
ans =
log(t+1)
```

4)

```
>> clear; syms x y f; f=sin(y)/(x^2*y+1);
>> int(f,x,-inf,inf)
ans =
pi*sin(y)/y^(1/2)
```

5)

```
>> clear; syms x y f; f=sin(y)/(x^2*y+1);
>> int(int(f,x,-inf,inf),-inf,inf)
ans =
0
```

9.

```
>> clear; syms x f; f=x^2+1;
>> f_int_diff=int(diff(f))
f _int_diff=
x^2
>> f_diff_int=diff(int(f))
f_diff_int =
x^2+1
```

diff(int(f)) is not equal to int(diff(f)), the difference between them is the constant item C.

10.

1)

```
>> clear; syms k n;
>> simplify(symsum(k^3,1,n))
ans =
1/4*n^4+1/2*n^3+1/4*n^2
```

2)

```
>> clear; syms k;
>> symsum(1/(k^2-1),1,inf)
ans =
sum(1/(k^2-1),k = 1 .. Inf)
```

3)

```
>> clear; syms k;
>> symsum(1/(k^2-1),2,inf)
ans =
3/4
```

4)

```
>> clear; syms k x f; f=k^2*x^k;
>> symsum(f,k,1,inf)
ans =
-x*(x+1)/(x-1)^3
```

11.

1)

```
>> clear; syms x f; f=exp(2*x);
>> taylor(f,15)
 ans =
```

```
1+2*x+2*x^2+4/3*x^3+2/3*x^4+4/15*x^5+4/45*x^6+8/315*x^7+2/315*x^8+4/2835*x^9+4/14
175*x^10+8/155925*x^11+4/467775*x^12+8/6081075*x^13+8/42567525*x^14
```

2)

```
>> clear; syms x f; f=exp(2*x);
>> taylor(f,9,-1)
ans =
exp(-2)+2*exp(-2)*(x+1)+2*exp(-2)*(x+1)^2+4/3*exp(-2)*(x+1)^3+2/3*exp(-2)*(x+1)^4+4/
15*exp(-2)*(x+1)^5+4/45*exp(-2)*(x+1)^6+8/315*exp(-2)*(x+1)^7+2/315*exp(-2)*(x+1)^8
```

3)

```
>> clear; syms x y f; f=exp(2*x*y);
>> taylor(f,5,x)
ans =
  1+2*x*y+2*y^2*x^2+4/3*y^3*x^3+2/3*y^4*x^4
```

12.

```
>> clear; format short e; syms a x; f=cos(x)+2*x; f_int=int(f);
>> b=[a+10*pi, a+5*pi, a+pi, a+1/2*pi, a+1/16*pi, a+1/1024*pi];
>> for k=1:6
y(k)=int(f,a,b(k));
y_app(k)=(subs(f_int, b(k))-subs(f_int, a))*(b(k)-a);
end
>> y, y_app
y =
  [                                                    20*a*pi+100*pi^2,
-2*sin(a)+10*a*pi+25*pi^2,                            -2*sin(a)+2*a*pi+pi^2,
cos(a)+a*pi+1/4*pi^2-sin(a),                sin(a+1/16*pi)+1/8*a*pi+1/256*pi^2-sin(a),
sin(a+1/1024*pi)+1/512*a*pi+1/1048576*pi^2-sin(a)]
y_app =
  [                                                    10*((a+10*pi)^2-a^2)*pi,
5*(-2*sin(a)+(a+5*pi)^2-a^2)*pi,
(-2*sin(a)+(a+pi)^2-a^2)*pi,                  1/2*(cos(a)+(a+1/2*pi)^2-sin(a)-a^2)*pi,
1/16*(sin(a+1/16*pi)+(a+1/16*pi)^2-sin(a)-a^2)*pi,
1/1024*(sin(a+1/1024*pi)+(a+1/1024*pi)^2-sin(a)-a^2)*pi]
>> y_0=subs(y, 0); y_app_0=subs(y_app, 0); error_y=y_0-y_app_0
error_y =
-3.0019e+004 -3.6290e+003 -2.1137e+001 -1.9792e+000   1.8777e-001   3.0679e-003
```

Conclusion: While b closes to a, $\left(\sin\left(\frac{a+b}{2}\right)+2\frac{(a+b)}{2}\right)(b-a)$ approaches to $\int_a^b(\cos x+2x)dx$.

13.

```
>> clear; format long e; syms x xi; f=1/(x+1)^2;
>> f_int=int(f, 0, 1); xi= subs(finverse(f),f_int)
```

```
Warning: finverse(1/(x+1)^2) is not unique.
> In C:\MATLAB6p5\toolbox\symbolic\@sym\finverse.m at line 43
  xi =
  2^(1/2)-1                                   % xi is in (0, 1)
>> abs(subs(f, xi)-f_int)
ans =

0                                             % ∫_0^1 1/(x+1)^2 dx = 1/(ξ+1)^2
```