MATLAB Exercise 7 – Calculus

1. Try to express y to be a compose function of x.

   1) $y1=\sqrt{1+u^2}, u=e^{-x}$;

   2) $y2=\sqrt{1+u^2}, u=\ln v, v=e^{-x}$;

   3) $y=\sqrt{1+u^2}, u=\ln v, v=\sin w, w=e^{-x}$.

2. Try to generate the converse function of y.

   1) $y=\sqrt{1+\ln^2 \sin x}$

   2) $y=\sqrt{x+\ln \sin u}$, where u is as the independent variable

   3) $y=\sqrt{x+\ln \sin u}$, where x is as the independent variable

3. Let $x=\frac{\pi}{4}$. Try to evaluate the value of $\sin x$ (express the result as 1/2*2^(1/2)) and $arc\sin(\sin x)$ (express the result as pi/4).

4. Compute the following limits, and simplify them.

   1) $\lim\limits_{x\to 0}\frac{\tan nx-\sin mx}{x}$ 2) $\lim\limits_{x\to y}\frac{e^x-e^y}{x-y}$ 3) $\lim\limits_{x\to+\infty}\frac{x^3}{2x+100}$

   4) $\lim\limits_{x\to-\infty}\frac{x^3}{\sin x}$ 5) $\lim\limits_{x\to\frac{\pi}{4}^+}(\tan x)^{\tan 2x}$ 6) $\lim\limits_{x\to\pi^-}\tan\frac{x}{2}$

5. 1) Compute the limit $\lim\limits_{h\to 0}\frac{(x+h)^n-x^n}{h}$, and simplify it.

   2) Compute $(x^n)'$ by diff.

6. Compute the following derivatives

   1) $\left.\frac{dg}{dx}\right|_{x=0}$ for $g(x)=\frac{x^3-5}{2x^2+7}$ 2) $\left.\frac{d^2g}{dxdy}\right|_{x=1}$ for $g(x,y)=\frac{x^3y-5y}{2x^2+7}$

   3) $\left.\frac{dg}{dy}\right|_{x=1,y=2}$ for $g(x,y)=\frac{x^3y-5y}{2x^2+7}$ 4) $f^{(5)}$ for $f=\sin x\sin 2x\sin 3x$

7. Let $A=[a_1,a_2,...,a_n]$ is a vector with *n* elements, say $A=[1,5,8,-2,6,3]$, how can we generate a new vector

   1) $B=[a_1-a_2,a_2-a_3,...,a_{n-1}-a_n]$?

2) $C=[a_1-2a_2+a_3, a_2-2a_3+a_4, ..., a_{n-2}-2a_{n-1}+a_n]$.

Help Select Matlab Help in the toolbar, then select Index and input **diff** to see its different usage.

8. Calculate the following calculus

1) $\int \frac{1}{x+1}dx$ 2) $\int_0^1 \frac{1}{x+1}dx$ 3) $\int_0^t \frac{1}{x+1}dx$

4) $\int_{-\infty}^{+\infty} \frac{\sin y}{(x^2y+1)}dx$ 5) $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\sin y}{(x^2y+1)}dxdy$.

Help Select Matlab Help in the toolbar, then select Index and input **int** to know the usage of this function , for example: int(f,x,-inf,inf)

9. Let $f=x^2+1$, compare it with the results of int(diff(f)) and diff(int(f)), respectively.

10. Compute the following summations

1) $\sum_{k=1}^{n} k^3$ 2) $\sum_{k=1}^{\infty} \frac{1}{k^2-1}$ 3) $\sum_{k=2}^{\infty} \frac{1}{k^2-1}$ 4) $\sum_{k=1}^{\infty} k^2x^k$

11. Evaluate Taylor series expansions of

1) $f(x)=e^{2x}$ at point 0 to the first 15 items;

2) $f(x)=e^{2x}$ at point -1 to the first 9 items;

3) $f(x)=e^{2xy}$ the first 5 items of Taylor series expansion responding to x.

12. *Compare the result $\int_a^b (\cos x+2x)dx$ with $\left(\sin\left(\frac{a+b}{2}\right)+2\frac{(a+b)}{2}\right)(b-a)$ when b equals to $a+10\pi$, $a+5\pi$, $a+\pi$, $a+1/2\pi$, $a+1/64\pi$, $a+1/256\pi$, respectively. What conclusion you may reach?

13. *Examine integral mean-value theorem, that is for any $f(x)\in C[a,b]$, there is a $\xi\in(a,b)$, such that $\int_a^b f(x)dx=f(\xi)(b-a)$. For example, try to find out the $\xi\in(0,1)$, such that $\int_0^1 \frac{1}{(x+1)^2}dx=\frac{1}{(\xi+1)^2}$.