

Key to MATLAB Exercise 4 – Eigenvalue

1.

1)

$$>> A=[3 2;3 -2]; [v,d]=eig(A)$$
 $v =$

$$\begin{matrix} 0.8944 & -0.3162 \\ 0.4472 & 0.9487 \end{matrix}$$
 $d =$

$$\begin{matrix} 4 & 0 \\ 0 & -3 \end{matrix}$$
 $>> \det(v)$ $ans =$

$$0.9899$$

The eigenvalues of A are 4 and -3, and the corresponding eigenspaces are

$V_4 = \{a (0.8944, 0.4472)^\top | \text{ for any } a \in \mathbb{R}\}$ and $V_{-3} = \{a (-0.3162, 0.9487)^\top | \text{ for any } a \in \mathbb{R}\}$.

A isn't defective, i.e. A is diagonalizable, because the determinant of v is not zero.

The keys to 2) to 9) are omitted.

2.

 $>> A=[2 0 0;0 4 0;1 0 2]; B=[2 0 0;0 4 0;-3 6 2];$ $>> eig(A)$ $ans =$

$$\begin{matrix} 2 \\ 2 \\ 4 \end{matrix}$$
 $>> eig(B)$ $ans =$

$$\begin{matrix} 2 \\ 2 \\ 4 \end{matrix}$$

Notice As 2 are multiple eigenvalues, we can't judge whether A is similar to B only by their eigenvalues. That is, we may say A is similar to B if A and B have the same Jordan normal forms.

 $>> [XA, JordanA] = jordan(A)$ $XA =$

$$\begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$$
 $JordanA =$

$$\begin{matrix} 4 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{matrix}$$
 $>> [XB, JordanB] = jordan(B)$ $XB =$

$$\begin{matrix} 0 & 0 & 2 \end{matrix}$$

```

1 0 0
3 -6 -3

```

JordanB =

```

4 0 0
0 2 1
0 0 2

```

A is similar to B because JordanA is equal to JordanB.

Or

>> [VA, DA]=eig(A)

VA =

```

0 0.0000 0
0 0 1.0000
1.0000 -1.0000 0

```

DA =

```

2 0 0
0 2 0
0 0 4

```

>> [VB, DB]=eig(B)

VB =

```

0 0.0000 0
0 0 0.3162
1.0000 1.0000 0.9487

```

DB =

```

2 0 0
0 2 0
0 0 4

```

>> rank(VA)==rank(VB)

ans =

1

2, 2, 4 are the eigenvalues of A and B, and the rank of VA equals that of VB, which means that A has the same Jordan normal form as B. Therefore A is similar to B.

3.

>> A=rand(4);

>> P=poly(A)

P =

```

1.0000 -1.5430 0.0898 -0.0310 0.1878

```

It means that the characteristic polynomial is $x^4 - 1.543x^3 + 0.0898x^2 - 0.031x + 0.1878$

>> roots(P)

ans =

```

1.4313
0.5970
-0.2427 + 0.4011i
-0.2427 - 0.4011i

```

The eigenvalues (or characteristic roots) of A are 1.4313, 0.5970, -0.2427+0.4011i and

-0.2427 - 0.4011i.

4.

1)

```
>> A=[0 1;1 0]; [X1, D]=eig(A)
X1=
-0.7071    0.7071
 0.7071    0.7071
D =
-1      0
 0      1
>> rank(X1)==size(X1, 1)
ans =
1
```

The answer is $X1 = \begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}$, $D = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. Here $X1$ is an invertible matrix.

We may check $X1$ by other functions such as `det`, `rref` and so on.

Or

```
>> [X2, S]=schur(A)
X2 =
-0.7071    0.7071
 0.7071    0.7071
S =
-1      0
 0      1
>> X2*S*inv(X2)
ans =
 0      1.0000
 1.0000      0
```

Therefore $X2$ is the answer, too.

Or

```
>> [X3,J]=jordan(A)
X3 =
 0.5000    0.5000
 -0.5000    0.5000
J =
-1      0
 0      1
>> X3*J*inv(X3)
ans =
 0      1
 1      0
```

Therefore $X3$ is also the answer.

The keys to 2) to 6) are omitted.

5.

1)
 $\gg A=[2\ 1;-2\ 1]; [v, d]=eig(A); B= v*sqrt(d)* inv(v)$
 $B =$

| | |
|---------|--------|
| 1.5119 | 0.3780 |
| -0.7559 | 1.1339 |

$\gg B*B$
 $ans =$

| | |
|---------|--------|
| 2.0000 | 1.0000 |
| -2.0000 | 1.0000 |

The key to 2) is omitted.

6.

$\gg \text{format rational}; A=[0\ 1\ -1;1\ 2\ 0;-1\ 0\ 3];$
 $\gg [U, T]=\text{schur}(A)$
 $U =$

| | | |
|------------|-----------|------------|
| -755/833 | 1661/6691 | -1573/4601 |
| 1573/4601 | 755/833 | -1661/6691 |
| -1661/6691 | 1573/4601 | 755/833 |

$T =$

| | | |
|-----------|-----------|----------|
| -655/1006 | 0 | 0 |
| 0 | 2665/1172 | 0 |
| 0 | 0 | 2874/851 |

7.

$\gg A=[1\ 1;2\ 3;1\ 0];$
 $\gg [u, s, v]=\text{svd}(A)$
 $u =$

| | | |
|---------|---------|---------|
| -0.3583 | 0.2312 | -0.9045 |
| -0.9208 | -0.2474 | 0.3015 |
| -0.1541 | 0.9409 | 0.3015 |

$s =$

| | |
|--------|--------|
| 3.9090 | 0 |
| 0 | 0.8485 |
| 0 | 0 |

$v =$

| | |
|---------|---------|
| -0.6022 | 0.7983 |
| -0.7983 | -0.6022 |

$\gg \text{SingularValuesOfA} = \text{diag}(s)$
 $\text{SingularValuesOfA} =$

| |
|--------|
| 3.9090 |
| 0.8485 |

$\gg \text{RootOfAA}=\text{roots}(\text{poly}(A^*A))$
 $\text{RootOfAA} =$

| |
|---------|
| 15.2801 |
| 0.7199 |

```
>> abs(SingularValuesOfA.^2-RootOfAA) < 10^(-10)
ans =
1
1
```

The singular values of A are 3.9090, 0.8485, their square equal those of the characteristic roots of $A'A$. That is, the singular values of A equal the square roots of the eigenvalues of $A'A$.

Try to use `SingularValuesOfA.^2==RootOfAA`, and explain the result.

8.

```
>> A=[-4 3 12; -17 -11 0; 1 12 3]; PolyA=poly(A); roots(PolyA)
```

```
ans =
```

```
-16.8781
2.4391 +10.6951i
2.4391 -10.6951i
```

```
>> eig(A)
```

```
ans =
```

```
-16.8781
2.4391 +10.6951i
2.4391 -10.6951i
```

```
>> schur(A)
```

```
ans =
```

| | | |
|----------|----------|----------|
| -16.8781 | 2.1204 | -12.4708 |
| 0 | 2.4391 | 7.7919 |
| 0 | -14.6799 | 2.4391 |

```
>> svd(A)
```

```
ans =
```

```
21.8740
14.6390
6.3427
```

```
>> svd(A)^2
```

```
ans =
```

```
478.4705
214.3003
40.2293
```

```
>> eig(A'*A)
```

```
ans =
```

```
40.2293
214.3003
478.4705
```

The above show that we may use functions `eig` and `roots` to calculate the eigenvalues of a matrix. The result of function `schur` is in real, that is, it will not show the eigenvalues of A if there are complex eigenvalues. And `svd` provides the values which are the square roots of the eigenvalues of $A'A$.

9.

```
>> A1=rand(4); B1=A1+A1'; % generate a symmetric matrix B1
```

```
>> A2=10*rand(4); B2=A2*A2'; % generate a symmetric matrix B2
```

We omitted the following for it is similar to that of Ex10.

10.

```
>> P=fix(10*rand(3));  
>> D1=diag([0, 1, 2]); D2=diag(1:3); Jordan1=[1 0 0; 0 2 1; 0 0 2];  
>> SingularMatrix=P*D1; % for D1 is a singular matrix  
>> NotSingularMatrix=P*D2;  
>> DiagonalMatrix=P' *P; % any symmetric matrix is diagonal.  
>> [Q r]=qr(P);  
>> NotDiagonalMatrix=Q'*Jordan1*Q;
```

11. Omitted