Key to MATLAB Exercise 4 – Eigenvalue

1.

1)

```
>> A=[3 2;3 -2]; [v,d]=eig(A)
v =
    0.8944   -0.3162
    0.4472    0.9487
d =
     4     0
     0    -3
>> det(v)
ans =
    0.9899
```

The eigenvalues of A are 4 and -3, and the corresponding eigenspaces are

V_4={a (0.8944,0.4472)'| for any a in R} and V_{-3}= {a (-0.3162, 0.9487)'| for any a in R}.

A isn't defective, i.e. A is diagonalizable, because the determinant of v is not zero.

The keys to 2) to 9) are omitted.

2.

```
>> A=[2 0 0;0 4 0;1 0 2]; B=[2 0 0;0 4 0;-3 6 2];
>> eig(A)
ans =
     2
     2
     4
>> eig(B)
ans =
     2
     2
     4
```

Notice As 2 are multiple eigenvalues, we can't judge whether A is similar to B only by their eigenvalues. That is, we may say A is similar to B if A and B have the same Jordan normal forms.

```
>> [XA, JordanA] = jordan(A)
XA =
     0     0     1
     1     0     0
     0     1     0
JordanA =
     4     0     0
     0     2     1
     0     0     2
>> [XB, JordanB] = jordan(B)
XB =
     0     0     2
```

```
     1     0     0
     3    -6    -3
JordanB =
     4     0     0
     0     2     1
     0     0     2
```

A is similar to B because JordanA is equal to JordanB.

Or

```
>> [VA, DA]=eig(A)
VA =
         0    0.0000         0
         0         0    1.0000
    1.0000   -1.0000         0
DA =
     2     0     0
     0     2     0
     0     0     4
>> [VB, DB]=eig(B)
VB =
         0    0.0000         0
         0         0    0.3162
    1.0000    1.0000    0.9487
DB =
     2     0     0
     0     2     0
     0     0     4
>> rank(VA)==rank(VB)
ans =
     1
```

2, 2, 4 are the eigenvalues of A and B, and the rank of VA equals that of VB, which means that A has the same Jordan normal form as B. Therefore A is similar to B.

3.

```
>> A=rand(4);
>> P=poly(A)
P =
1.0000   -1.5430    0.0898   -0.0310    0.1878
```

It means that the characteristic polynomial is $x^4 - 1.543x^3 + 0.0898x^2 - 0.031x + 0.1878$

```
>> roots(P)
ans =
    1.4313
    0.5970
   -0.2427 + 0.4011i
   -0.2427 - 0.4011i
```

The eigenvalues (or characteristic roots) of A are 1.4313, 0.5970. -0.2427+0.4011i and

-0.2427 - 0.4011i.

4.

1)

```
>> A=[0 1;1 0]; [X1, D]=eig(A)
X1=
   -0.7071    0.7071
    0.7071    0.7071
D =
    -1     0
     0     1
>> rank(X1)==size(X1, 1)
ans =
     1
```

The answer is $X1=\begin{pmatrix}-0.7071 & 0.7071\\ 0.7071 & 0.7071\end{pmatrix}$, $D=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$. Here X1 is an invertible matrix.

We may check X1 by other functions such as det, rref and so on.

Or

```
>> [X2, S]=schur(A)
X2 =
   -0.7071    0.7071
    0.7071    0.7071
S =
    -1     0
     0     1
>> X2*S*inv(X2)
ans =
         0    1.0000
    1.0000         0
```

Therefore X2 is the answer, too.

Or

```
>> [X3,J]=jordan(A)
X3 =
    0.5000    0.5000
   -0.5000    0.5000
J =
    -1     0
     0     1
>> X3*J*inv(X3)
ans =
     0     1
     1     0
```

Therefore X3 is also the answer.

The keys to 2) to 6) are omitted.

5.

1)

```
>> A=[2 1;-2 1]；[v, d]=eig(A)；B= v*sqrt(d)* inv(v)
B =
    1.5119    0.3780
   -0.7559    1.1339
>> B*B
ans =
    2.0000    1.0000
   -2.0000    1.0000
```

The key to 2) is omitted.

6.

```
>> format rational; A=[0 1 -1;1 2 0;-1 0 3];
>> [U, T]=schur(A)
U =
   -755/833     1661/6691    -1573/4601
   1573/4601     755/833     -1661/6691
  -1661/6691    1573/4601     755/833
T =
   -655/1006        0            0
       0         2665/1172       0
       0            0         2874/851
```

7.

```
>> A=[1 1;2 3;1 0];
>> [u, s, v]=svd(A)
u =
   -0.3583    0.2312   -0.9045
   -0.9208   -0.2474    0.3015
   -0.1541    0.9409    0.3015
s =
    3.9090         0
         0    0.8485
         0         0
v =
   -0.6022    0.7983
   -0.7983   -0.6022
>> SingularValuesOfA = diag(s)
SingularValuesOfA =
    3.9090
    0.8485
>> RootOfAA=roots(poly(A'*A))
RootOfAA =
   15.2801
    0.7199
```

```
>> abs(SingularValuesOfA.^2-RootOfAA) < 10^(-10)
ans =
     1
     1
```

The singular values of A are 3.9090, 0.8485, their square equal those of the characteristic roots of A'A. That is, the singular values of A equal the square roots of the engenvalues of A'A.

Try to use SingularValuesOfA.^2==RootOfAA, and explain the result.

8.

```
>> A=[-4 3 12; -17 -11 0; 1 12 3]; PolyA=poly(A); roots(PolyA)
ans =
 -16.8781
   2.4391 +10.6951i
   2.4391 -10.6951i
>> eig(A)
ans =
 -16.8781
   2.4391 +10.6951i
   2.4391 -10.6951i
>> schur(A)
ans =
  -16.8781    2.1204  -12.4708
         0    2.4391    7.7919
         0  -14.6799    2.4391
>> svd(A)
ans =
   21.8740
   14.6390
    6.3427
>> svd(A)^2
ans =
  478.4705
  214.3003
   40.2293
>> eig(A'*A)
ans =
   40.2293
  214.3003
  478.4705
```

The above show that we may use functions eig and roots to calculate the eigenvalues of a matrix. The result of function schur is in real, that is, it will not show the eigenvalues of A if there are complex eigenvalues. And svd provides the values which are the square roots of the eigenvalues of A'A.

9.

```
>> A1=rand(4); B1=A1+A1';          % generate a symmetric matrix B1
```

```
>> A2=10*rand(4); B2=A2*A2';        % generate a symmetric matrix B2
```

We omitted the following for it is similar to that of Ex10.

10.

```
>> P=fix(10*rand(3));
>> D1=diag([0, 1, 2]); D2=diag(1:3); Jordan1=[1 0 0; 0 2 1; 0 0 2];
>> SingularMatrix=P*D1;                    % for D1 is a singular matrix
>> NotSingularMatrix=P*D2;
>> DiagonalMatrix=P' *P;                   % any symmetric matrix is diagonal.
>> [Q r]=qr(P);
>> NotDiagonalMatrix=Q'*Jordan1*Q;
```

11. Omitted