

MATLAB Exercise 4 – Eigenvalue

1. Find the eigenvalues and the corresponding eigenspaces for each of the following matrices. And judge whether it is a defective matrix.

1) $\begin{pmatrix} 3 & 2 \\ 3 & -2 \end{pmatrix}$

2) $\begin{pmatrix} 6 & -4 \\ 3 & -1 \end{pmatrix}$

3) $\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$

4) $\begin{pmatrix} 3 & -8 \\ 2 & 3 \end{pmatrix}$

5) $\begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}$

6) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

7) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

8) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & -1 \end{pmatrix}$

9) $\begin{pmatrix} 4 & -5 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$

Help Select **Matlab Help** in the toolbar, then select **Index** and input **eig**, distinguish the difference usage of this function. For example **[V,D] = eig(A)** and **[V,D] = eig(A,B)**.

2. Judge whether the following two matrix are similar?

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 1 & 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 2 \end{pmatrix}$$

3. Use **poly** and **roots** function to compute the characteristic polynomial and characteristic roots of a random 4×4 matrix. According to the result show that the characteristic polynomial and characteristic roots of A in mathematics formula.
4. In each of the following, factor the matrix A into a product $XD X^{-1}$, where D is diagonal. (use two different methods)

1) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

2) $A = \begin{pmatrix} 5 & 6 \\ -2 & -2 \end{pmatrix}$

3) $A = \begin{pmatrix} 2 & -8 \\ 1 & -4 \end{pmatrix}$

4) $A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 3 \\ 1 & 1 & -1 \end{pmatrix}$

5) $A = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$

6) $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{pmatrix}$

5. For each of the following, find a matrix B such that $B^2 = A$.

1) $A = \begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix}$

2) $A = \begin{pmatrix} 9 & -5 & 3 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

6. Given $A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix}$, find an orthogonal matrix U that diagonalizes A . Please display

the results in rational format. (**schur**)

Help Select **Matlab Help** in the toolbar, then select **Index** and input **schur**, distinguish the difference usage of this function. For example $T = \text{schur}(A)$; $[U, T] = \text{schur}(A)$.

7. Let $A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 0 \end{pmatrix}$. Compute the singular values and the singular value decomposition of A .

Compare square of the singular values of A with the eigenvalues of $A'A$. Are they the same?

Help Select **Matlab Help** in the toolbar, then select **Index** and input **svd** to know the usage of this function.

8. Let $A = \begin{pmatrix} -4 & 3 & 12 \\ -17 & -11 & 0 \\ 1 & 12 & 3 \end{pmatrix}$. Compute the eigenvalues of A by **roots** and **eig** functions

respectively. Compute its eigenvalue decomposition, Schur decomposition and singular value decomposition respectively. Compare the results and show the differences.

9. Please generate 4 symmetric matrices and check the following proposition “If the eigenvalues of a symmetric matrix A are $\lambda_1, \lambda_2, \dots, \lambda_n$, then the singular values of A are $|\lambda_1|, |\lambda_2|, \dots, |\lambda_n|$ ”.

10. Please generate 10 matrices (some of them are singular 奇异, others are nonsingular 非奇异 即可逆, others are diagonalized matrices and others are not diagonalized matrices) and calculate their eigenvalues and singular values. Then count the number of nonzero eigenvalues and singular values. Show the conclusion you guess.

11. *Compute the singular value decomposition of matrix

$$A = \begin{pmatrix} 2 & 5 & 4 \\ 6 & 3 & 0 \\ 6 & 3 & 0 \\ 2 & 5 & 4 \end{pmatrix}.$$

- 1) Use the singular value decomposition to find orthonormal bases for $R(A')$ and $N(A)$
- 2) Use the singular value decomposition to find orthonormal bases for $R(A)$ and $N(A')$