

## Key to MATLAB Exercise 2 - Solving Linear Systems of Equations

1. 

```
>> A=round(10*rand(5))
>> B=round(20*rand(5))-10
```

  - a) 

```
>> det(A)          ans =5972
>> det(A')          ans = 5972          Yes
```
  - b) 

```
>> det(A+B)          ans = 36495
>> det(A)+det(B)      ans = 26384          No
```
  - c) 

```
>> det(A*B)          ans = 121900464
>> det(A)*det(B)      ans = 121900464      Yes
```
  - d) 

```
>> det(A')*det(B')    ans = 121900464
>> det(A'*B')         ans = 121900464      Yes
```
  - e) 

```
>> det(inv(A))        ans = 1.6745e-004
>> inv(det(A))        ans = 1.6745e-004      Yes
```
  - f) 

```
>> det(A*inv(B))      ans = 0.2926
>> det(A)*inv(det(B)) ans = 0.2926          Yes
```
2. 

```
>> A=round(10*rand(6)).
```

  - a) 

```
>> A=round(10*rand(6)); B=A; B(2,:)=A(1,:); B(1,:)=A(2,:);
>> det(A)            ans = 4636
>> det(B)            ans = -4636
```

Interchanging two rows of a matrix changes the sign of the determinant.
  - b) 

```
>> C=A; C(3,:)=4*A(3,:)
>> det(C)            ans = 18544
>> det(A)*4          ans = 18544
```

Multiplying a single row of a matrix by a scalar has the effect of multiplying the value of the determinant by that scalar.
  - c) 

```
>> D=A; D(5,:)=A(5,:)+2*A(4,:)
>> det(A)            ans =4636
>> det(D)            ans=4636
```

Adding a multiple of one row to another does not change the value of the determinant.
3.
  - a) 

```
>> A=[1 2; 2 -2]; b=[4;2]; x=A\b
x =
     2
     1
>> A*x
ans =
     4
     2
>> det(A)    ans =-6
```

Or

```
>> A=[1 2; 2 -2]; b=[4;2]; x=inv(A)*b
```

Or

```
>> A=[1 2; 2 -2]; b=[4;2]; c=[A b]; d=rref(c); x=d(:, end)
```

Same does below. Omitted.

b) 

```
>> A=[2 3; 5 1]; b=[-1;4]; x=A\b
```

```
x =
```

```
1
```

```
-1
```

```
>> A*x
```

```
ans =
```

```
-1
```

```
4
```

```
>> det(a)
```

```
ans = -13
```

c) 

```
>> A=[4 2 -1;3 -1 2;11 3 0]; b=[2;10;8]; x=A\b
```

Warning: Matrix is close to singular or badly scaled.

Results may be inaccurate. RCOND = 5.139921e-018.

```
x =
```

```
1.0e+016 *
```

```
-0.4053
```

```
1.4862
```

```
1.3511
```

```
>> A*x
```

```
ans =
```

```
0
```

```
8
```

```
8
```

```
>> det(a)
```

```
ans = 0
```

It isn't the solution, because  $\det(a)=0$ .

d) 

```
>> A=[1 3 1; 2 1 1; -2 2 -1]; b=[1;5;-8]; x=A\b
```

```
x =
```

```
2
```

```
-1
```

```
2
```

```
>> A*x
```

```
ans =
```

```
1
```

```
5
```

```
-8
```

```
>> det(A)
```

```
ans =
```

```
3
```

4.

```
>> A=[1 2 3; 2 3 4; 5 4 6]; b=[0;0;1];
```

```
>> format rational
```

```
>> A1=det([b A(:, 2:3)]); A2=det([A(:, 1) b A(:, 3)]); A3=det([A(:, 1:2) b]); ADet=det(A);
>> x=[A1/ADet; A2/ADet; A3/ADet]
x =
    1/3
   -2/3
    1/3
```

5.

```
>> AdjOfA=det(A)*inv(A)
AdjOfA =
```

```

    2         0        -1
    8        -9         2
   -7         6        -1
```

6.

```
a) >> A=[2 1 -3; 4 5 1; -2 -1 4]; b=[0;8;2];
```

```
>> x=A\b
```

```
x =
```

```

    4
   -2
    2
```

```
>> c=null(A,'r')
```

```
c =
```

```
Empty matrix: 3-by-0
```

So x is the solution.

or

```
>> d=rref([A b])
```

```
d =
```

```

    1     0     0     4
    0     1     0    -2
    0     0     1     2
```

```
>> x=d(:,end)
```

```
x =
```

```

    4
   -2
    2
```

```
b) >> A=[2 1 -3; 4 5 1; 2 4 4]; b=[0;8;8];
```

```
>> c=null(A,'r')
```

```
c =
```

```

    8/3
   -7/3
    1
```

```
>> x0=A\b
```

```
Warning: Matrix is singular to working precision.
```

```
x0 =
```

```

-4/3
 8/3
 0
>> syms k
>> x=x0+c*k
x =
-4/3+8/3*k1
 8/3-7/3*k1
      k1
or
>> d=rref([A b])
d=
      1      0      -8/3      -4/3
      0      1       7/3       8/3
      0      0       0        0
>> syms k
>> x=d(:,end)+[-d(1:end-1,end-1);1]*k
x =
[ -4/3+8/3*k]
[  8/3-7/3*k]
[           k]

```

7.

```

>> format short
>> A=[2 1 -3; 4 5 1; 2 -2 -10]; d=rref(A)
d =
 1.0000      0 -2.6667
      0  1.0000  2.3333
      0      0      0
>> i=rank(A)
i=
 2
>> x=[-d(1:i,i+1);1]
x =
 2.6667
-2.3333
 1.0000
>> A*x
ans =
 1.0e-014 *
-0.0444
-0.1776
      0

```

8. Omitted

9. Omitted

10.

```
>> A=[1 2 1; 2 4 2; 2 1 1]; B=[3 1 2; 1 2 2; 3 1 4]; C=A+i*B
```

```
1).C =
```

```
1.0000 + 3.0000i    2.0000 + 1.0000i    1.0000 + 2.0000i
2.0000 + 1.0000i    4.0000 + 2.0000i    2.0000 + 2.0000i
2.0000 + 3.0000i    1.0000 + 1.0000i    1.0000 + 4.0000i
```

```
>> C'
```

```
ans =
```

```
1.0000 - 3.0000i    2.0000 - 1.0000i    2.0000 - 3.0000i
2.0000 - 1.0000i    4.0000 - 2.0000i    1.0000 - 1.0000i
1.0000 - 2.0000i    2.0000 - 2.0000i    1.0000 - 4.0000i
```

```
>> C.'
```

```
ans =
```

```
1.0000 + 3.0000i    2.0000 + 1.0000i    2.0000 + 3.0000i
2.0000 + 1.0000i    4.0000 + 2.0000i    1.0000 + 1.0000i
1.0000 + 2.0000i    2.0000 + 2.0000i    1.0000 + 4.0000i
```

The first matrix is the complex conjugate transpose of C while the second one is the transpose of C. Because the elements of C are complex, the results are different.

```
2). >> A
```

```
A =
```

```
1    2    1
2    4    2
2    1    1
```

```
>> A'
```

```
ans =
```

```
1    2    2
2    4    1
1    2    1
```

```
>> A.'
```

```
ans =
```

```
1    2    2
2    4    1
1    2    1
```

The first matrix is equal to the second one.

```
3). >> inv(A)
```

Warning: Matrix is singular to working precision.

```
ans =
```

```
Inf    Inf    Inf
Inf    Inf    Inf
Inf    Inf    Inf
```

```
>> pinv(A)
```

```
ans =
```

```
-0.0727    -0.1455    0.6364
0.1273     0.2545   -0.3636
```

0.0182    0.0364    0.0909

The first matrix is not equal to the second one.

4). `>> inv(B)`

ans =

0.6000	-0.2000	-0.2000
0.2000	0.6000	-0.4000
-0.5000	0	0.5000

`>> pinv(B)`

ans =

0.6000	-0.2000	-0.2000
0.2000	0.6000	-0.4000
-0.5000	-0.0000	0.5000

Because B is not singular, `inv(B)` is equal to `pinv(B)`.

11. Omitted