MATLAB Lecture 6 – Polynomial

多项式

Ref: MATLAB→Mathematics→Polynomials and Interpolation

- **Vocabulary:**

polynomial 多项式

root 根	arithmetic operation 算术运算
multiply 乘法	divide 除法
derivative 导数	differentiation 微分法
evaluation 求值	partial-fraction 部分分式
expansion 展开	convolution 卷积
product 乘积	deconvolution 去卷积
quotient 商	remainder 余项
multiple roots 重根	direct 直接的
term 项	transfer function 转换函数，传递函数

- **Some functions**

conv deconv poly polyder polyval polyvalm roots * residue * polyfit

- **Polynomials**

✧ **Representing Polynomials**

MATLAB represents polynomials as row vectors containing coefficients ordered by descending powers.

```
>> p = [1 0 -2 -5];          % represents x^3 - 2x - 5

p =
     1     0    -2    -5
>> sym_p = poly2sym(p)   % represents a polynomial in sym form
sym_p =
x^3-2*x-5
```

✧ **Create Polynomials**

```
>> p = [1 0 -2 -5]          % represents a polynomail x^3 - 2x - 5

p =
     1     0    -2    -5

>> r = [0, 1, -1]; poly(r)    %generate a polynomial x(x-1)(x+1) ,whose roots are 0,1,-1

ans =
     1     0    -1     0
>> a = [1 2; 3 4]; poly(a)   % generate the characteristic polynomials of matrix…
```

$$\begin{pmatrix} \lambda-1 & -2 \\ -3 & \lambda-4 \end{pmatrix}, \text{ i.e. } \begin{vmatrix} \lambda-1 & -2 \\ -3 & \lambda-4 \end{vmatrix} = \lambda^2 - 5\lambda - 2$$

```
ans =
    1.0000    -5.0000    -2.0000
```

✧ Polynomial Evaluation

```
>> polyval(p,5)          % evaluates a polynomial at a specified value, say 5.
ans =
   110
>> subs(sym_p,5)         % substitute the sym variable x in sym_p with 5
ans =
   110

>> X = [2 4 5; -1 0 3; 7 1 5];
>> Y = polyvalm(p, X)   % create a square matrix X and evaluate the polynomial p at X…
```

$$Y = X^3 - 2X - 5I$$

```
Y =
   377   179   439
   111    81   136
   490   253   639
```

✧ Polynomial Roots

```
>> r = roots(p)          % calculates the roots of a polynomial p
r =
     2.0946
    -1.0473 +      1.1359i
    -1.0473 -      1.1359i
```

✧ Polynomial Arithmetic operation

Addition

```
>>p2 = [0 2 -1 3]; add_p = p+p2      %calculates sum of two polynomials p and p2. …
                                       Here the matrix dimensions must agree.
add_p =
     1     2    -3    -2
>> p3 = poly2sym(p2); add_p_sym = sym_p + p3     %sym_p pulses p3 and display the …
                                                   result in sym form.
add_p_sym =
x^3-3*x-2+2*x^2
>> sym2poly(add_p_sym)          % returns a row vector containing the coefficients …
                                   of the symbolic polynomial P
ans =
     1     2    -3    -2
```

Subtraction (Omit. It is similar to addition)

Multiplication (Correspond to the operations convolution)

```
>> a = [1 2]; b = [2 0 -1]; c = conv(a, b); poly2sym(c)   …
```

% compute the product of $(x+2)(2x^2-1)$

```
ans =
2*x^3+4*x^2-x-2
```

Division (Correspond to the operations convolution and deconvolution)

```
>> [q, r] = deconv(c, a)          %dividing c by a is quotient q and remainder r
q =
     2     0    -1
r =
     0     0     0     0
```

✧ Polynomial Derivatives

```
>> q = polyder(p)                 %computes the derivative of polynomial (x^3 - 2x - 5)'
q =
     3     0    -2
>> a = [1 3 5]; b = [2 4 6]; c = polyder(a,b)     %computes the derivative of the …
```

product of two polynomials $[(x^2+3x+5)(2x^2+4x+6)]'$

```
c =
     8    30    56    38
>> [q,d] = polyder(a,b)     %computes the derivative of the quotient of two polynomials…
```

$$\left[\frac{(x^2+3x+5)}{(2x^2+4x+6)}\right]' = \frac{q(x)}{d(x)}$$

```
q =
    -2    -8    -2
d =
     4    16    40    48    36
```

✧ *Partial Fraction Expansion

residue finds the partial fraction expansion of the ratio of two polynomials. This is particularly useful for applications that represent systems in transfer function form. For polynomials *b* and *a*, if there are no multiple roots,

$$\frac{b(x)}{a(x)} = \frac{r_1}{x-p_1} + \frac{r_2}{x-p_2} + \ldots + \frac{r_n}{x-p_n} + k_s$$

where *r* is a column vector of residues, *p* is a column vector of pole locations, and *k* is a row vector of direct terms.

```
>> b = [-4 8]; a = [1 6 8]; [r, p, k] = residue(b, a)
```

% $\frac{-4x-8}{x^2+6x+8} = \frac{-12}{x+4} + \frac{8}{x+2}$

```
r =
   -12
     8
```

```
p =
     -4
     -2
k =
     [ ]
```

Given three input arguments (r, p, and k), residue converts back to polynomial form.

```
>> [b2, a2] = residue(r, p, k)
```

$$\% \frac{-12}{x+4} + \frac{8}{x+2} = \frac{-4x-8}{x^2+6x+8}$$

```
b2 =
     -4     8
a2 =
      1     6     8
```

✧ Polynomial Function Summary

Function	Description
conv	Multiply polynomials.
deconv	Divide polynomials.
poly	Polynomial with specified roots.
polyder	Polynomial derivative.
polyval	Polynomial evaluation.
polyvalm	Matrix polynomial evaluation.
roots	Find polynomial roots.
residue	Partial-fraction expansion (residues).
polyfit	Polynomial curve fitting.